

まずは次を計算しておく。

$$\mu_X = E[X] = \frac{1}{2} \times 2 + \frac{1}{2} \times (-2) = 0$$

$$\mu_Y = E[Y] = \frac{1}{2} \times 2 + \frac{1}{2} \times (-2) = 0$$

$$\text{Var}[X] = \frac{1}{2} \times (2-0)^2 + \frac{1}{2} \times (-2-0)^2 = 4$$

$$\text{Var}[Y] = \frac{1}{2} \times (2-0)^2 + \frac{1}{2} \times (-2-0)^2 = 4$$

$$\sigma_X = \sqrt{4} = 2, \quad \sigma_Y = \sqrt{4} = 2$$

$$\begin{aligned} \textcircled{1} \quad \text{Cov}[X, Y] &= \frac{1}{4} (2-0)(2-0) + \frac{1}{4} (-2-0)(2-0) \\ &\quad + \frac{1}{4} (2-0)(-2-0) + \frac{1}{4} (-2-0)(-2-0) \\ &= 0 \end{aligned}$$

$$\rho_{XY} = \frac{0}{2 \times 2} = 0$$

$$\begin{aligned} \textcircled{2} \quad \text{Cov}[X, Y] &= \frac{3}{8} (2-0)(2-0) + \frac{1}{8} (-2-0)(2-0) \\ &\quad + \frac{1}{8} (2-0)(-2-0) + \frac{3}{8} (-2-0)(-2-0) \\ &= 2 \end{aligned}$$

$$\rho_{XY} = \frac{2}{2 \times 2} = \frac{1}{2} = 0.5$$

$$\begin{aligned} \textcircled{3} \quad \text{Cov}[X, Y] &= \frac{1}{8} (2-0)(2-0) + \frac{3}{8} (-2-0)(2-0) \\ &\quad + \frac{3}{8} (2-0)(-2-0) + \frac{1}{8} (-2-0)(-2-0) \\ &= -2 \end{aligned}$$

$$\rho_{XY} = \frac{-2}{2 \times 2} = -\frac{1}{2} = -0.5$$